Superluminal behavior in wave propagation: a famous case study in the microwave region

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Abstract - The meaning of superluminality in electromagnetic wave propagation is addressed. The interesting case of diffractionless beams is analysed in view of their very peculiar properties with particular attention to their possible superluminality. A case study in the microwave region, reported in a famous paper by an Italian team of researchers [Phys. Rev. Lett. 84, 4830 (2000)], is analysed and interpreted both in terms of ray optic arguments and by considering the specific beam shape of that experiment, which can be regarded as a Bessel-Gauss type, thus exhibiting a fake group velocity higher than c when the observation is limited to the beam propagation axis.

1. Introduction

Superluminality in electromagnetic (e.m.) wave propagation has been always thought to by physicists as a forbidden obsession or a demonic dream. As a matter of fact, superluminal behavior has been observed in evanescent wave propagation and interpreted both within a near field field propagation effect for an electric dipole antenna and by means of a phase time model of the tunneling time theory [1]. It is not an easy task to find an e.m. beam capable to exhibit some kind of superluminal features and the question is certainly always debated in each case whether the basic principle of Special Relativity that nor energy neither information can propagate faster than light was violated or not. So-called diffractionless beams and X-waves have gained a considerable interest also because of their superluminal behaviour. Essentially, these types of solutions of the e.m. wave equation do not undergo phase variation under propagation. This property, which gives them the name of "diffractionless" turns out to be very useful, for instance, in nonlinear optics applications [2-5]. X-waves [6-8] and

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nonlinear X-waves [9] have been studied and uccessfully employed due to their ability, as nonspreading wavepacket solutions of the linear and nonlinear wave equation respectivley, to carry unchanged images with no diffraction-induced distortion over long distances. For example, by using acoustic X-waves attractig applications in medical ultrasonic imaging are possible [6]. Bessel beams, which are typical diffractionless solutions of the wave equations though carrying infinite energy, and their physical counterpart that carries finite energy, i.e. the Bessel-Gauss beams, are also drawning great attention lately as they are eigenstates of the orbital angular momentum of light and as such are used in a huge number of novel applications [10, 11]. Superluminality, however, has been the most attractive property amongst all of the above beams both for theoretical implications [12, 13] and for practical consequences such as a possible use in signal transmission. Several schemes have been proposed to experimentally realize X-waves and Bessel beams [14, 15] even in the optical domain [16]. Saari *et al*. [14] realized a wideband, nonspreading axissymmetric Bessel-X-pulse and reconstructed the spatiotemporal profile of the field by using a very smart idea. In fact, time-integration of the interference patterns of the field in the radial plane, at successive points along the propagation axis allows one to fully reconstruct the Bessel-like shape of the generated beam. In doing so they took advantage of the superluminal velocity of the Bessel pulse. The authors of [14], however, clearly state that the superluminality of their propagation-invariant interence pattern is a purely geometric effect.

Here we focus rather on a more intriguing and very much debated experiment performed in the microwave ($\lambda \approx 3.5$ cm) [15]. Mugnai et al. [15] shaped a microwave Bessel-like beam and found it to propagate faster than light along the optical axis. They places a circular slit right at the exit of a horn located at the focal plane of a spherical reflector. In the end they measured the arrival times of the microwave pulses at a movable receiver antenna connected to a detector. The experiment was carried out in free space so to rule out all the possible effects due to dispersion of the propagation medium. A number of papers, in fact, evidenced how some superluminal behaviour can arise from different phenomena occurring in the propagation medium [17 -24]. The results reported in [15] have been analyzed in [25, 26] where it is shown that on-axis superluminal propagation velocity of the field peak cannot be referred to as the group velocity of the wavepacket. Here we first dedicate a section to a simple ray optics argument that explains the superluminal behaviour observed in [15] as a bias in the way to measure the propagation velocity of the beam. A more accurate analyses of the scheme implemented in [15] is illustrated in sec. three, where we consider a Bessel-Gauss beam and show that the on-axis projection of the beam wavefront propagation velocity is superluminal. Conclusions are drawn in the last section.

2. Geometric effect

The geometric configuration of the experimental setup used in [15] is schematized in the inset of Fig. 1 a). In the polar frame with origin in the curvature centre O of the mirror **M**, a point-like source P radiates towards **M**. In terms of rays, a ray hits **M** in A and, after reflection, reaches Q. Each ray is associated to a given ϕ while *L* and the effective optical path travelled by the ray, Λ = AP + AQ, read rescpectively:

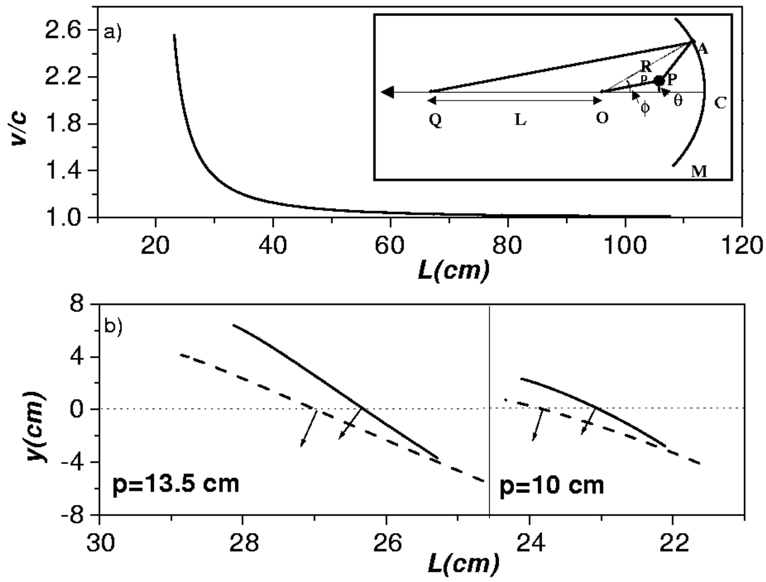

FIG. 1. a) INSET: Scheme of the experimental setup used in [15] in order to shape a Bessel beam in the microwave domain. Behaviour of the effective propagation velocity, *v*/*c*, versus *L* for p = 13.5 cm, θ = 0.35 rad, and R = 25 cm, namely the same values of the parameters used in [15]. b) Sketch of the beam wavefronts in proximity of the optical axis, for p = 13.5 cm and 10 cm, and for θ = 16° (solid line) and θ = 24° (dashed line), i.e. the same θ values considered in [15].

$$L(\phi) = \frac{\mathrm{p}\cdot\sin(\phi-\theta)}{\sin\phi-\frac{\mathrm{p}}{\mathrm{R}}\sin(2\phi-\theta)} \tag{1a}$$

$$\frac{\Lambda(\phi)}{\sqrt{\mathrm{R}^2-2\mathrm{pR}\cdot\cos(\phi-\theta)+\mathrm{p}^2}} = \frac{2\sin\phi-\frac{\mathrm{p}}{\mathrm{R}}\sin(2\phi-\theta)}{\sin\phi-\frac{\mathrm{p}}{\mathrm{R}}\sin(2\phi-\theta)} \tag{1b}$$

R being the radius of curvature of **M** and p = OP. The quantity $\frac{\Delta L}{\Delta\phi}/\frac{\Delta\Lambda}{\Delta\phi}$ gives the effective propagation velocity of the microwave beam normalized to *c*, *v*/*c*, as measured in [15]. This quantity is plotted in Fig.1 a) versus *L*. Very interestingly, the plot reproduces the experimental results reported in [15] for p = 13.5 cm and θ = 0.35 rad, thus confirming a substantial conceptual flaw of the authors of [15] in interpreting the measured velocity, i.e. the Bessel beam on-axis crest velocity as the propagation velocity of the e.m. wave which is rather the group velocity. Figure 1 b) represents the beam wavefront in proximity of the optical axis for the two rays corresponding to θ = 16° and 24° (p = 13.5 cm and 10 cm) which are the two cases considered in [15], by placing circular slits of different diameters.
It is also worth stressing that for the rays reflected by the upper part of **M** ($\phi \geq 0$) Q approaches O while *L* becomes smaller and smaller, whereas for those reflected by the lower part of **M** ($\phi \leq 0$) Q gets far away and $v/c \rightarrow 1$ in good agreement with the quoted experimental results.

3. Propagation velocity of Bessel-Gauss beams

In order to provide an explanation for the superluminal effect reported in [15] which was deeper than ray optics but moved towards the same concept we performed an analysis of the field in the focal region of the apparatur realized by Mugnai *et al*. To this end we have considered a wavepacket constituted by monochromatic Bessel-Gauss fields which resembles a pure Bessel beam, but carries a finite amount of energy. Such a field of frequency $\omega + \Omega$, Ω being the carrier, confocal parameter *b*, and unit amplitude at focus reads as:

$$u(\rho, z; \omega) = \frac{w_0}{w(z)} e^{i((\omega+\Omega)\tau + i\psi(z))} J_0\left(i\frac{(\omega+\Omega)b\sin\theta}{cq(z)}\rho\right) \qquad (2)$$

with J_0 the Bessel function, $q(z) = z - ib/2$, $\psi(z) = \arctan(2z/b)$, $w_0/w(z) = b/\sqrt{4z^2 + b^2}$ and

$$\tau = t - \frac{z}{c}\cos\theta + \frac{\rho^2 + z^2\sin^2\theta}{2cq(z)} \qquad (3)$$

A generally complex time. The time delay $t - \frac{z}{c}\cos\theta$ in Eq.(3) represents a wave travelling at the superluminal velocity $c/\cos\theta$. Far apart from the focal region the field in Eq.(2) takes a conical shape of aperture θ. A Gaussian wavepacket of the above modes, centred at frequency Ω, is represented by an integral of $u(\rho,z;\omega)$ in the deviation frequency ω weighted by the generally complex function $F(\omega) =$

$\exp\left(-\frac{\omega^2}{2\sigma^2}\right)$. Inside the focal spot, by Fourier transforming back from the frequency to the time domain, $u(\rho,z;t)$ can be shown to read:

$$u(\rho,z;t) = \frac{w_0}{w(z)} e^{i\Omega\tau + i\psi(z) - (\sigma\tau)^2 + \frac{\varsigma^2}{2}} \times \left[I_0\left(\frac{\varsigma^2}{2}\right) J_0(\alpha) + 2\sum_1^{\infty} (-i)^m I_m\left(\frac{\varsigma^2}{2}\right) J_{2m}(\alpha) \right] \quad (4)$$

where $\alpha = \left(\frac{\Omega}{\sigma} + 2i\sigma\tau\right)\varsigma$, $\varsigma = -i\frac{b\sin\theta}{q(z)}\frac{\sigma}{c}$ and I_m represents the modified Bessel functions of the first kind. Since inside the focal spot $|\zeta| \ll 1$ the expansion (4) reduces to:

$$|u(\rho,z;t)| \approx \frac{w_0}{w(z)} \left| e^{i\Omega\tau - (\sigma\tau)^2} I_0\left(\frac{\varsigma^2}{2}\right) J_0\left[\left(\frac{\Omega}{\sigma} + 2i\sigma\tau\right)\varsigma\right] \right|. \quad (5)$$

In particular, along the z-axis $u(0,z;t)$ reads:

$$u(0,z;t) = \frac{w_0}{w(z)} e^{i\Omega\tau + i\psi(z) - (\sigma\tau)^2} \quad (6)$$

That is the wavepacket peaked in correspondence of the coordinates t, z obeys the propagation equation:

$$ct = z\left(\cos\theta + \frac{z^2}{z^2 + b^2}\frac{\sin^2\theta}{2}\right) \quad (7)$$

and seems to travel in proximity of the focus along the z-axis at a superluminal velocity which depends on the θ angle. As already pointed out this superluminality just concerns the projection of the field along the z-axis, whereas in order to calculate either the phase or the group velocity of the beam the entire wavepacket must be considered.

4. Conclusion

In conclusion, we have presented a twofold argument to explain the observed superluminality in the microwave range [15] in terms of a flaw in the concept and the measurement of the propagation velocity of an electromagnetic wave used by Mugnai *et al*. Both in terms of ray optics and in terms of a near-field analysis of the Bessel beam created by Mugnai *et al*. With both arguments we have demonstrated that the observed superluminality, far from being a mistake of the experiment, is rather a mistake in the interpretation of the measured quantity. The results reported by Mugnai *et al*. are not at all in conflict with causality. One can simply interpret them as demonstrating that, by looking along the beam propagation direction, light can actually propagate from point A to point B faster than *c*. There is certainly no violation of causality as the light at B is not causally connected to the light at A. i.e. the light at A cannot be regarded as the source of the light at B. It is a lot like the famous example of the rotating flashlight producing a spot on a distant wall that moves faster than *c*: the spot at the point A is not the source of the spot at subsequent point B.

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